O.P. JINDAL SCHOOL, PATRATU
MODEL QUESTIONS

Class: XII | M.M.: 80

Subject: Mathematics | Duration: 180 Min.

General Instructions-

- All questions are compulsory.
- There are internal choices in some questions.
- This question paper contains- five sections A, B, C, D and E.
- Section-A has 20 questions of 1 mark each.
- Section -B has 05 questions of 2 marks each.
- Section-C has 06 questions of 3 marks each.
- Section-D has 03 case-study based questions of 4 marks each.
- Section -E has 04 questions of 5 marks each.

SECTION-A

Choose the correct option.

1. The difference of the order and the degree of the given differential equation is:

$$\sqrt{x+\left(\frac{dy}{dx}\right)^2} = a\left(\frac{d^2y}{dx^2}\right)^{\frac{1}{3}}$$

(a) -1 (b) 0
(c) 2 (d) 1

2. If $3x + 2y = siny$, then $\frac{dy}{dx}$ is:

(a) $\frac{3}{\cos y-2}$ (b) $\frac{\sin y-1}{2}$
(c) $\frac{2-\sin y}{3}$ (d) $\frac{2-\cos y}{3}$

3. The value of $\int_0^{\frac{\pi}{2}} e^x(\sin x + \cos x)dx$ is

(a) e (b) $e^{\frac{\pi}{2}}$
(c) $e^{\frac{\pi}{2}-1}$ (d) e^2

4. Which of the following relations is symmetric but neither reflexive nor transitive for a set $A = \{1, 2, 3\}$

(a) $R = \{(1,2),(1,3),(1,4)\}$ (b) $R = \{(1,2),(2,1)\}$
(c) $R = \{(1,1),(2,2),(3,3)\}$ (d) $R = \{(1,1),(1,2),(2,3)\}$

5. The integrating factor of $\frac{dy}{dx} + y = \frac{1+y}{x}$ is

(a) $\frac{e^x}{x}$ (b) $\frac{e^{-x}}{x}$
(c) xe^x (d) x^2e^x

6. If $A = \begin{bmatrix} cos\alpha & -sin\alpha \\ sin\alpha & cos\alpha \end{bmatrix}$, then $A + A' = I$, then value of α is:

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{3}$
(c) π (d) $\frac{3\pi}{2}$

7. If $\begin{bmatrix} 3c+6 & a-d \\ a+d & 2-3b \end{bmatrix} = \begin{bmatrix} 12 & 2 \\ -8 & -4 \end{bmatrix}$ are equal, then value of $ab - cd$ is

(a) 4 (b) 16
(c) -4 (d) -16

8. The interval in which the function f given by $f(x) = x^2 e^{-x}$ is strictly increasing, is:

(a) $(-\infty, \infty)$ (b) $(-\infty, 0)$

(c) $(2, \infty)$ (d) $(0, 2)$

9. If $\begin{vmatrix} 2 & 3 & 2 \\ x & x & x \\ 4 & 9 & 1 \end{vmatrix} + 3 = 0$, then the value of x is:

(a) 3 (b) 0

(c) -1 (d) 1

10. If $A = \begin{bmatrix} 2 & \lambda & -3 \\ 0 & 2 & 4 \\ 1 & 1 & 3 \end{bmatrix}$, then A^{-1} exists if:

(a) $\lambda = 2$ (b) $\lambda \neq 2.5$

(c) $\lambda \neq -2.5$ (d) $\lambda \neq 2$

11. If $f(x) = x \tan^{-1} x\, , then\ f'(1) =$

(a) $1 + \frac{\pi}{4}$ (b) $\frac{1}{2} + \frac{\pi}{4}$

(c) $\frac{1}{2} - \frac{\pi}{4}$ (d) 2

12. The value of $\boldsymbol{\lambda}$ such that the vector $\vec{a} = 2\hat{\imath} + \lambda\hat{\jmath} + \hat{k}$ and $\vec{b} = \hat{\imath} + 2\hat{\jmath} + 3\hat{k}$ are orthogonal is:

(a) 3/2 (b) -5/2

(c) -1/2 (d) 1/2

13. The cofactor of element a_{23} in the matrix $\begin{bmatrix} 1 & 0 & 4 \\ 3 & 5 & -1 \\ 0 & 1 & 2 \end{bmatrix}$ is:

(a) 1 (b) 2

(c) -1 (d) 0

14. For any vector $\vec{a}$, the value of $|\vec{a}.\hat{\imath}|^2 + |\vec{a}.\hat{\jmath}|^2 + |\vec{a}.\hat{k}|^2$ is:

(a) a (b) a^2

(c) 1 (d) 0

15. A set of values of decision variables that satisfies the linear constraints and non-negativity conditions of an L.P.P. is called its

(a) Unbounded solution (b) optimum solution

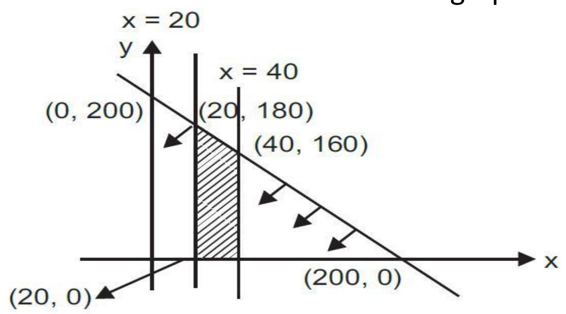
(c) Feasible solution (d) None of these

16. A problem in Mathematics is given to three students whose chances of solving it are $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$, respectively. If the events of their solving the problem are independent then the probability that the problem will be solved, is:

(a) $\frac{1}{4}$ (b) $\frac{1}{3}$

(c) $\frac{1}{2}$ (d) $\frac{3}{4}$

17. For an L.P.P. the objective function is $Z = 400x + 300y$, and the feasible region determined by a set of constraints is shown in the graph.



Find the coordinates at which the objective function is maximum.

(a) (20, 0) (b) (40, 0)

(c) (40, 160) (d) (20, 180)

18. What is the angle between vectors $\vec{a}$ and $\vec{b}$, if $|\vec{a}| = 1, |\vec{b}| = 2$ and $\vec{a} \times \vec{b} = \hat{\imath} + \hat{\jmath} + \hat{k}$?

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{3}$

(c) $\frac{2\pi}{3}$ (d) $\frac{\pi}{6}$

ASSERTION-REASON BASED QUESTIONS

In the following questions, a statement of assertion(A) is followed by a statement of Reason(R). Choose the correct answer out of the following choices.

(a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true and R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false but R is true.

19. **Assertion(A):** Lines $\frac{x+2}{-2} = \frac{y-1}{3} = \frac{z-2}{1}$ and $\frac{x-3}{-3} = \frac{y}{-2} = \frac{z+1}{2}$ are coplanar.

Reason(R): Let line l_1 passes through the point (x_1, y_1, z_1) and parallel to the vector whose direction ratios are $a_1, b_1 and\ c_1$; and let line l_2 passes through the point (x_2, y_2, z_2) and parallel to the vector whose direction ratios are $a_2, b_2 and\ c_2$.

Then both lines l_1 and l_2 are coplanar if and only if $\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0$

20. **Assertion(A):** The area of a parallelogram with diagonals $\vec{a}$ and $\vec{b}$ is $\frac{1}{2}|\vec{a} \times \vec{b}|$.

Reason(R): If $\vec{a}$ and $\vec{b}$ represents the adjacent sides of a triangle, then area of triangle can be obtained by evaluating $|\vec{a} \times \vec{b}|$.

SECTION-B

21. Express $\tan^{-1}\left(\frac{\cos x}{1-\sin x}\right), -\frac{3\pi}{2} < x < \frac{\pi}{2}$ in simplest form.

22. Show that the function $f(x) = x^3 - 3x^2 + 6x - 100$ is increasing on R.

23. Evaluate: $\int_{-1}^{1} \frac{|x|}{x} dx$

OR

Find: $\int \frac{x^2+2}{x^2+1}\, dx$

24. Show that the function $f: R \to R$ given by $f(x) = 4x^3 + 7$ for all $x \to R$ is bijective.

25. Find the intervals in which the function $f(x) = 3x^4 - 4x^3 - 12x^2 + 5$ is

(a) Strictly increasing (b) Strictly decreasing

SECTION-C

26. Find $\frac{dy}{dx}$, when $x = a(t + sint)$ and $y = a(1 - cost)$.

27. Solve the following differential equation: $\frac{dy}{dx} = x^3 cosec\ y$, given that $y(0) = 0$.

OR

Solve the following differential equation:

$\left[x\, sin^2\left(\frac{y}{x}\right) - y\right] dx + xdy = 0$

28. Solve the following Linear Programming Problem graphically:

Maximise$Z = 8x + 9y$ subject to the constraints:

$2x + 3y \le 6, 3x - 2y \le 6, y \le 1; x, y \ge 0$

29. Find: $\int \frac{x^2}{(x-1)(x+1)^2}\, dx$

30. Evaluate: $\int_0^{\frac{\pi}{2}} \frac{dx}{1+\sqrt{\tan x}}$

31. The random variable X has a probability distribution P(X) of the following form, where ‘k’ is some real number:

$$P(X) = \begin{cases} k, if\ x = 0 \\ 2k, if\ x = 1 \\ 3k, if\ x = 2 \\ 0, otherwise \end{cases}$$

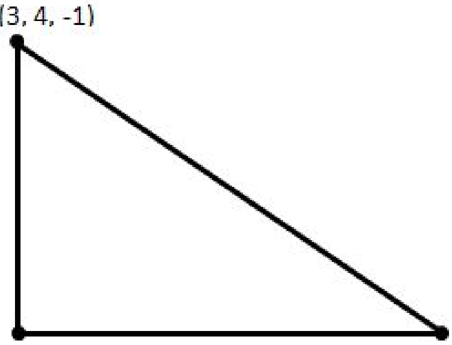
(a) Determine the value of k.
(b) Find $P(X < 2)$
(c) Find $P(X > 2)$

SECTION-D

32. Sonam wants to prepare a sweet box for Diwali at home. For making lower part of box, she takes a square piece of cardboard of side 18 cm.
Now, x cm be the length of each side of the square cardboard which is to be cut off from corner of the square piece of side 18 cm.

Based on the above information, answer the following questions:
(i) Express Volume of the open box formed by folding up the cutting corner in terms of x and find the value of x for which $\frac{dv}{dx} = 0$.
(ii) Sonam is interested in maximising the volume of the box. So, what should be the side of the square to be cut off so that the volume of the box is maximum?

33. Answer the questions based on the given information:
The flight path of two airplanes in a flight simulator game are shown below. The coordinates of the airports P and Q are given.



Airplane 1 flies directly from P to Q. Airplane 2 has a layover at R and then flies to Q. The path of Airplane 2 from P to R can be represented by the vector $5\hat{\imath} + \hat{\jmath} - 2\hat{k}$.
(i) Find the vector that represents the flight path of Airplane 1. Show your steps.
(ii) Write the vector representing the path of Airplane 2 from R to Q. Show your steps.

(iii) What is the angle between the flight paths of Airplane 1 and Airplane 2 just after take-off? Show your work.

OR

(iii) Consider that Airplane 1 started the flight with a full fuel tank. Find the position vector of the point where a third of the fuel runs out if the entire fuel is required for the flight. Show your work.

34. Read the following passage and answer the questions given below:

There are two antiaircraft guns, named as A and B. The probabilities that the shell fired from them hits an airplane are 0.3 and 0.2 respectively. Both of them fired one shell at an airplane at the same time.

(i) What is the probability that the shell fired from exactly one of them hit the plane?

(ii) If it is known that the shell fired from exactly one of them hit the plane, then what is the probability that it was fired from B?

SECTION-E

35. Using integration, find the smaller area enclosed by the circle $x^2 + y^2 = 4$ and the line $x + y = 2$.

36.. Find the vector equation of the line through the point $(1, 2, -4)$ and perpendicular to the two lines $\vec{r} = (8\hat{\imath} - 19\hat{\jmath} + 10\hat{k}) + \lambda(3\hat{\imath} - 16\hat{\jmath} + 7\hat{k})$
and $\vec{r} = (15\hat{\imath} + 29\hat{\jmath} + 5\hat{k}) + \mu(3\hat{\imath} + 8\hat{\jmath} - 5\hat{k})$.

OR

Find the shortest distance between the following lines:
$l_1: \vec{r} = \hat{\imath} + 2\hat{\jmath} - 4\hat{k} + \lambda(2\hat{\imath} + 3\hat{\jmath} + 6\hat{k})$
$l_2: \vec{r} = 3\hat{\imath} + 3\hat{\jmath} - 5\hat{k} + \mu(4\hat{\imath} + 6\hat{\jmath} + 12\hat{k})$

37. If $A = \begin{bmatrix} 1 & 2 & -3 \\ 3 & 2 & -2 \\ 2 & -1 & 1 \end{bmatrix}$, then find A^{-1} and use it to solve the following system of the equations:
$x + 2y - 3z = 6, 3x + 2y - 2z = 3, 2x - y + z = 2$

38. A function $f: R - \{-1, 1\} \to R$ is defined by $f(x) = \frac{x}{x^2-1}$

(a) Check if f is one-one.

(b) Check if f is onto.

Show your work.

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